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Interplay of Localization and Superconducting Fluctuations above the Critical Point

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A review of some developments in localization effects on conductivity and magnetoconductivity is given. The determination of inelastic scattering rates for electrons in thin disordered metallic films is emphasized. In two-dimensional disordered superconductors above T_c, the superconducting fluctuations play an essential role. Recent work on the interplay of localization and superconducting fluctuation effects in determining the magnetoconductivity and the inelastic rate is described.

KEY WORDS:

It is a great honor to be able to contribute to the I. M. Lifshitz Memorial Volume of the *Journal of Statistical Physics*. In view of I. M. Lifshitz's many fundamental and pioneering works on the electronic properties of disordered materials, it is appropriate here to describe some current research in the area of what has become called "localization." In particular, the interplay of localization effects and superconductivity has become a subject of intensive experimental investigation; this has stimulated renewed theoretical interest in the problem of "dirty superconductors" which was discussed so extensively 20 years ago.

In this communication, after a brief resumé of relevant recent developments in localization, we shall focus on the determination of the inelastic lifetime of electrons in disordered metallic and superconducting systems. We restrict our discussion to two dimensions, where the localization effects are most pronounced.

The basic localization problem ("Anderson localization"), concerns the behavior of noninteracting electrons in a disordered solid. Later we shall

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introduce some features of the electron–electron interaction perturbatively. In 1976, F. J. Wegner[1] proposed the beginnings of a scaling theory for electrons in disordered systems. In 1979, the scaling theory was put on a firm basis[2,3] and several important results were derived. Among these was the prediction[2] that in two dimensions there is no true metallic conductivity; at large enough scales (sample sizes), the conductivity must scale (eventually) exponentially to zero. In other words, there are no extended electronic states in a disordered two-dimensional system and for a large class of random potentials, all states are exponentially localized. The latter result, of course, has been known for some time for one dimension.

The depression of the conductivity due to the localization effect is caused by scattering processes which involve coherent backwards scattering of electron wave packets by the random potential.[4] This is expressed mathematically in terms of the diffusion propagator in the particle–particle channel[5] which in two dimensions gives a negative contribution to the conductivity which is logarithmically divergent but is cutoff by an appropriate length scale L:

$$\sigma = \sigma_0 - \frac{e^2}{\hbar\pi^2} \ln(L/l) \tag{1}$$

where σ_0 is the usual (Sommerfeld) conductivity and l is the elastic mean free path.

The length scale L which determines the logarithmic decrease of the conductivity can be determined by various factors. At absolute zero, at zero frequency, and in zero magnetic field, it is the sample size.[2] At finite temperature, inelastic scattering destroys the coherent scattering so L becomes L_T, which may be described as the distance an electron diffuses in the random potential before suffering an inelastic scattering.[6] The quantity L_T is given by

$$L_T = [l l_i(T)/2]^{1/2} \tag{2}$$

where $l_i(T)$ is the temperature-dependent, inelastic mean free path. Since l may be determined experimentally from σ_0 in Eq. (1), it appears possible to extract $l_i(T)$ and its T dependence from experiment. At finite magnetic field H, L becomes the magnetic length

$$L_H = (\hbar c/eH)^{1/2} \tag{3}$$

and a negative magnetoresistance is predicted[7] and confirmed experimentally.[2]

[2] For a large list of experimental results see Ref. 8.

The comparison with experiment is complicated by the fact that if the Coulomb interaction between electrons is included another contribution to the conductivity arises which is not present in the absence of disorder. It is due to the breakdown of translation invariance and the fact that electrons have an enhanced interaction due to their diffusive rather than ballistic, motion. If this effect is included perturbatively, an additional correction of precisely the form in Eq. (1) is obtained where L becomes $\hbar v_F/k_B T$.

It has proved possible to disentangle the localization and interaction processes by measuring the magnetoresistance for which the two effects behave quite differently. The former, as remarked above, acquires a $\ln H$ dependent negative magnetoresistance, the latter being essentially unaffected. In fact, the magnetoresistance turns out to be a universal function of L_T/L_H.[7,9] Therefore, magnetoresistance experiments yield the following: A confirmation of the universal prefactor of the logarithmic localization conductivity correction in Eq. (1), the temperature dependence of L_T, and hence the inelastic mean free path $l_i(T)$ and the magnitude of $l_i(T) = v_F \tau_i(T)$, where v_F is the Fermi velocity and $1/\tau_i$ the inelastic scattering rate.

The inelastic scattering rate in normal metallic films is determined by electron–electron scattering and electron–phonon scattering; the former dominates at temperatures below 1 K. The inelastic rates are strongly influenced by the presence of disorder. The electron–photon rate has been calculated by A. Schmid,[10] whose analysis is easily extended to two dimensions. The electron–electron rate in two dimensions was calculated by Abrahams, Anderson, Lee, and Ramakrishnan[11] and Lopes dos Santos.[12] A somewhat different derivation was given by Altshuler, Aronov, and Khmel'nitskii.[12] It was pointed out by Fukuyama and Abrahams[13] that the correct inelastic L_T is obtained not from the single-electron inelastic scattering rate but rather from a cutoff in the particle–particle diffusion propagator ("Cooperon"). This comes from a finite self-energy (mass) caused by inelastic scattering; it may be computed perturbatively. For the electron–electron and electron–phonon cases, the two methods give the same results.[11,13] At low temperature, the result for the temperature-dependence of the electron–electron inelastic rate is

$$1/\tau_i \propto T \ln A \tag{4}$$

The argument of the logarithm varies among the different calculations and is temperature dependent for the case of the cutoff of the zero wave number Cooperon. According to Fukuyama,[14] the magnetoresistance measures a certain average of $1/\tau_i(q)$ since a sum of Cooperons of different wave number q enters. In that case, the results of Ref. 12 appear to be appropriate for experimental comparisons. This form and the predicted[12–14] order of magnitude has been confirmd in a variety of experiments.[15–17]

Recently, similar sorts of experiments have been performed on two-dimensional metallic films which at low enough temperatures become superconducting. Now, the existence of superconducting fluctuations above T_c complicates the problem significantly. The conductivity is enhanced by the presence of superconducting fluctuations.[18,19] The application of a magnetic field suppresses[20] the fluctuations and leads to a positive contribution to the magnetoresistance.

There is an "anomalous' contribution to the conductivity originally discussed by Maki[21] and Thompson[22] which also is suppressed by a magnetic field. Larkin[23] has recently shown that the Maki–Thompson (MT) conductivity contribution exhibits a positive magnetoresistance with the same field dependence as the negative one of ordinary localization.[7] The Larkin–Maki–Thompson magnetoresistance depends on the electron inelastic lifetime, more precisely the mass of the Cooperon, in a similar manner to the ordinary negative localization magnetoresistance. The Cooperon mass is directly related[23] to the pair-breaking parameter[22] δ of the Maki–Thompson theory. As has been pointed out already in Refs. 24, the existence of superconducting fluctuations gives another contribution to the electron inelastic lifetime and hence to the diffusive Cooperon mass. The process is spontaneous creation and decay of superconducting fluctuations from and into electrons of opposite spin and momentum. It is to be expected that this contribution to the inelastic rate increases as $(T-T_c)^{-1}$ rather than decreasing as a power of T as is the case for electron–electron and electron–phonon processes.

Larkin's prediction[23] has been verified by several experimental groups[25] in the region of low fields and temperatures not too close to T_c. The limitations of Larkin's theory are

$$\ln(T/T_c) \gg 1/T\tau_i \tag{5a}$$

$$4DeH/c < T\ln(T/T_c) \tag{5b}$$

where H is the magnetic field, D the diffusion constant $D = v_F l/2$, and we have set Boltzmann's constant equal to unity. A second difficulty is that in the Larkin magnetoresistance expression, the cutoff of the Cooperon diffusion pole, whatever its source, is taken to be independent of magnetic field. This is correct for the electron–electron interaction contribution to the cutoff.[26] It is certainly not the case for the superconducting fluctuation contribution since the fluctuations are suppressed by a magnetic field.

In what follows, we describe how the limitations of the Larkin–Maki–Thompson magnetoresistance theory are overcome. Before doing so, we again mention briefly the other important magnetoresistance contributions. We have already mentioned the localization negative magnetoresistance which occurs whether or not the sample is superconducting. We also referred to the positive magnetoresistance due to the suppression of the superconducting fluctuation conductivity (Aslamazov–Larkin contribution).[18,19] This magnetoresistance was already discussed more than a decade ago[20] and it is only important very close to T_c. In this paper, we do not discuss the influence of spin-orbit impurity scattering which strongly modifies the magnetoresistence.[4] A number of other contributions which are important far from T_c have been discussed by Altshuler *et al.*[27] Specifically, we describe results for the region $\ln T/T_c < 1$.

The first problem involves extending the limits of Eqs. (5) for the magnetoresistance expression to be used in analyzing the experimental data. The limitations arise because Larkin[23] neglected a term of order $[T\tau_i \ln(T/T_c)]^{-1}$ resulting in Eq. (5a) and because he nglected the influence of the mgnetic field on the superconducting fluctuation resulting in Eq. (5b). The necessary extensions have been made recently by Lopes dos Santos and Abrahams.[28] The results of this work are first, that the zero field MT conductance is found for all T:

$$\sigma_{\mathrm{MT}}(T, H=0) = (e^2/2\pi^2 d)\,\beta(T, \tau_i)\ln\left[\frac{\ln(T/T_c)}{\delta}\right] \tag{6}$$

In Eq. (6), d is the film thickness and δ is the MT "pair-breaking" parameter, $\delta = \pi/8T\tau_i$. The function $\beta(T, \tau_i)$ is a generalization of the $\beta(T)$ introduced by Larkin.[23] The generalization removes the restriction of Eq. (5a). Here we give only the low-T form of $\beta(T, \tau_i)$:

$$\beta(T, \tau) = \left(\frac{\pi^2}{4}\right)\left(\ln\frac{T}{T_c} - \delta\right)^{-1} \tag{7}$$

Larkin's $\beta(T)$ does not contain the pair breaker δ which appears on the right-hand side of Eq. (7).

The second result of Ref. 28 arises by inclusion of the magnetic field in the superconducting fluctuation. It gives $\Delta\sigma_{\mathrm{MT}}(T, H)$, the MT magnetoconductance, in regions of T and H not previously discussed. Thus, at low fields

$$4DeH/c < \min[T\ln(T/T_c), T] \tag{8}$$

but at *all* $T > T_c$, the form of the Larkin results[23] is obtained but with the generalized $\beta(T, \tau_i)$[28] replacing his $\beta(T)$. Finally, close to $T_c[\ln(T)T_c \ll 1]$, $\Delta\sigma_{\mathrm{MT}}(T, H)$ has been obtained at higher fields than allowed by Eq. (8). We do not give the complete formulas[28] here. However, at high field, as the superconducting fluctuation becomes quenched, $\Delta\sigma_{\mathrm{MT}}(T, H)$ saturates at the value $-\sigma_{\mathrm{MT}}(T, H=0)$, a satisfying result.

As we have pointed out previously, the value of $1/\tau_i$ may itself depend on H, which will give an implicit field dependence of $\beta(T, \tau_i)$ as well as an extra field dependence in the field dependent terms of $\Delta\sigma_{\mathrm{MT}}(T, H)$. This will significantly complicate the reduction of experimental data. The field dependence of the superconducting fluctuation contribution to $1/\tau_i$, the Cooperon cutoff, has been worked out by Brenig, Chang, Abrahams, and Wölfle.[29] In the absence of magnetic field, the cutoff $\rightarrow \infty$ at $T = T_c$.[24] However, the magnetic field suppresses T_i and the divergence is shifted down to $T_c(H)$, but the results are really not valid for T too close to $T_c(H)$, namely, $\ln T/T_c$ must be larger than $(k_F l)^{-1/2}$, where k_F is the Fermi momentum. Unfortunately, there seems to be no simple way to disentangle the T, H dependences of $1/\tau_i$ from magnetoresistance data. Some procedural suggestions are made in Ref. 29.

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